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TEMPERATURES AND STRESSES DUE TO QUENCHING OF HOLLOW CYLINDERS

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After forging, gun tube blanks are heated to a high temperature and quenched to near room temperature before tempering to achieve the required material properties. The purpose of the quench is to bypass the knee of the pearlite phase. This program was undertaken to establish cooling curves while the material is being quenched and to compute the thermal and transformation stresses involved.

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The temperatures are computed using implicit finite difference schemes. The problem treated is a nonlinear one in radial heat flow. The problem with cylindrical geometry is assumed to be axisymmetric and the coefficients in the equation such as thermal conductivity are treated as functions of temperature. The boundary conditions are written in a general form allowing the use of temperature, convection or heat flux boundary conditions. The nonlinear problem is solved by using two finite difference schemes in tandem. The first computes the temperatures at the $n+\frac{1}{2}$ time step assuming constant coefficients computed from a previous temperature distribution. This generates a temperature distribution throughout the thickness which is used to compute new coefficients for the second finite difference scheme which calculates the temperature distribution at the $n+1$ time step. This process is continued until a steady state or some desired level is reached.

At each time step, the program computes the thermal stresses associated with the temperatures. In addition to this, when the temperature reaches a certain level, called martensite start (M_s), the material begins to undergo the martensite transformation. This transformation involves an increase in material volume of about 3%-4%. A simple view of these transformation stresses is taken and the stresses due to this volume change are computed as the temperature cools to below the martensite start temperature throughout the wall thickness.

Results are presented for various boundary conditions including those expected to exist in the quenching facility.

ACKNOWLEDGEMENT

The author wishes to acknowledge the assistance of Richard Haggerty and Royce Soanes in the computational phase of this work.

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I. INTRODUCTION. There are several techniques available for quenching metals. The object is to develop some desired microstructure in the material. In the Watervliet Arsenal's rotary forge facility, forged cylindrical tubes are heated to an austenitizing temperature of approximately 1550°F and then quenched to form the desired martensitic structure in the material. Both external and internal diameter quenches are utilized. The outside diameter is spray quenched with four water jets in a diametral plane spraying water onto the tube while the tube is rotating. There are several of these planes located along the axis. The bore or inside diameter is quenched by flushing through a nozzle located at one end of the tube.

While the facility was still in the development stage, several tubes (a higher incidence than normal) developed cracks and some of these were interpreted as quench cracks. While the problem was judged to be metallurgical in nature and has been settled, an interest was indicated in understanding the transient temperatures and stresses involved in the quenching problem and this led to the present study.

First, the transient temperature distribution of an axially symmetric hollow cylinder is found. Differences along the axis are assumed to be minor and ignored. The thermal properties are treated as functions of temperature rendering the equation for heat conduction as nonlinear. The finite difference method is used to solve the temperature problem. The Crank-Nicolson equation which is implicitly stable is used.

In the present study, the stresses due to the temperature distribution and the martensite transformation were computed, assuming the problem was elastic and linear. From the computed stresses, it was obvious that some plastic deformation must occur and that an elastic-plastic analysis was required. This work will be performed in a future study.

The stresses due to the transformation are assumed to be strictly due to a change in volume. As the steel transforms from the austenitic structure to a martensitic structure, a volume increase of 3%-4% occurs in the transformed material. This volume increase gives rise to transformation stresses.

The thermal problem and stress problem are treated as being uncoupled. The heat generated from the transformation is small and will have negligible effect on the temperature distribution during the described quenching procedure. The transformation begins when the temperature in the material reaches M_s, the martensite start temperature, and is completed when the material is past the M_f, or martensite finish temperature. Another technique used in quenching is to quench the material to M_s and slowly allow the transformation to take place. In this case, the heat generated during the transformation, might be significant and the coupled problem might need to be considered.

II. PROBLEM STATEMENT. The partial differential equation for temperatures in a hollow cylinder is

$$\frac{1}{r}\frac{\partial}{\partial r}\left(k(u)r\,\frac{\partial u}{\partial r}\right) = c(u)\rho(u)\,\frac{\partial u}{\partial t} \tag{1}$$

where r represents the distance along a radius, u the temperature, and t the time. The thermal conductivity, specific heat and density are represented by k, c and ρ respectively. These properties are assumed to be functions of the temperature. Axial symmetry is assumed and any effects along the axis are ignored.

The initial condition is given by

$$u(r,o) = U_o \tag{2}$$

where U_o would represent the high austenitizing temperature. The boundary conditions for the problem described would be of the convection type. However, to allow some flexibility in the program, they were written in the following form

$$\begin{aligned} \frac{\partial u}{\partial r} - h_1 u &= -g_1 \qquad \text{at } r = a \\ \frac{\partial u}{\partial r} - h_2 u &= -g_2 \qquad \text{at } r = b \end{aligned} \tag{3}$$

where r = a specifies the inside radius and r = b the outside radius of the cylinder. The values h_i and g_i can be varied at either surface so that various boundary conditions can be specified. For example, if $g_1 = 0$ at r = a, then h_1 is the Nusselt number and the

convection boundary condition is indicated at $r = a$. If h_2 and g_2 are very large but the ratio $g_2/h_2 = U_b$ then the temperature U_b is specified at $r = b$.

Since the thermal properties of the material must be considered as functions of temperature, the partial differential equation (1) is nonlinear and numerical techniques are needed to solve the problem. An implicit scheme based on the Crank-Nicolson equation was used in writing the finite difference scheme for the temperatures.

III. FINITE DIFFERENCE EQUATIONS. The Crank-Nicolson representation of Eq. (1) is

$$\frac{1}{(a+(i-\tfrac{1}{2})\Delta r)\Delta r}\left\{\frac{a+i\Delta r}{2} k_{i+\frac{1}{2},n+\frac{1}{2}}\delta_r(u_{i+\frac{1}{2},n+1} + u_{i+\frac{1}{2},n}) - \right.$$

$$\left.\frac{a+(i-1)\Delta r}{2} k_{i-\frac{1}{2},n+\frac{1}{2}}\delta_r(u_{i-\frac{1}{2},n+1} + u_{i-\frac{1}{2},n})\right\} = c_{i,n+\frac{1}{2}}\rho_{i,n+\frac{1}{2}} \frac{u_{i,n+1} - u_{i,n}}{\Delta t} \tag{4}$$

where i is the ith node
n is the time step
Δt is the time increment
Δr is the space increment

and

$$\delta_r u_{i+\frac{1}{2},n} = \frac{u_{i+1,n} - u_{i,n}}{\Delta r}$$

$$k_{i+\frac{1}{2},n+\frac{1}{2}} = k\left(\frac{u_{i+1,n+\frac{1}{2}} + u_{i,n+\frac{1}{2}}}{2}\right) \tag{5}$$

The finite difference equation (4) is written about the point r_i, $t_{n+\frac{1}{2}}$. If the temperature and its spacial derivatives can be written without requiring their values at $n+\frac{1}{2}$, then the equations become linear [1]. This is accomplished by arithmetic averaging the finite difference analogues at the points r_i, t_n and r_i, t_{n+1}, and the resulting analogue is the average of the forward and backward analogues. To solve these equations for the temperatures, it is required

1. von Rosenberg, D. U., Methods for the Numerical Solution of Differential Equations, American Elsevier Publishing Co., Inc., New York, 1969.

to know the properties at the n+½ time step. This will be shown later.

In writing the boundary conditions as Equations (3), it is necessary to locate the nodes as shown in Figure 1. There are no nodes located on the boundary about which finite difference equations are written. The boundary conditions are used to eliminate the 0th and R+1st nodes from the equations. The temperatures on the boundary are found from extrapolation or through the use of the boundary conditions after the spatial temperature distribution is found at that time step.

The values of the thermophysical properties at the ½ time step can be found through various projection methods [1]. The one chosen is the centered Taylor series projection. A set of equations similar to Equation (4) are written between the n and n+½ time step. Thus the values of the properties would thus be required at the n+¼ time level. The technique allows the computation to take place using properties evaluated at the known nth time level. Under those conditions, the equations are linear, the properties known and the temperatures at the n+½ time step can be found. Knowing this, new property values can be found and the equations solved for the temperatures at the n+1 time step.

An alternate technique which still arrives at the equivalent of Equation (4) was used for the centered Taylor series projection. Equation (1) is rewritten in following form.

$$k(u)\,\frac{\partial^2 u}{\partial r^2} + \frac{\partial k(u)}{\partial u}\left(\frac{\partial u}{\partial r}\right)^2 + \frac{k(u)}{r}\,\frac{\partial u}{\partial r} = \rho(u)c(u)\,\frac{\partial u}{\partial t} \tag{6}$$

Using the Crank-Nicolson finite difference analogue about the ith node and n+½ time step, Equation (6) becomes, after some rearranging,

$$\frac{1}{2(\Delta r)^2}\left[k(u_{i,n+\frac{1}{2}}) + k'(u_{i,n+\frac{1}{2}})\left[\frac{u_{i+1,n+\frac{1}{2}} - u_{i-1,n+\frac{1}{2}}}{2\Delta r}\right]\frac{\Delta r}{2}\right](u_{i+1,n+1} + u_{i+1,n})$$

$$+\frac{1}{2(\Delta r)^2}\left[k(u_{i,n+\frac{1}{2}}) - k'(u_{i,n+\frac{1}{2}})\left[\frac{u_{i+1,n+\frac{1}{2}} - u_{i-1,n+\frac{1}{2}}}{2\Delta r}\right]\frac{\Delta r}{2}\right](u_{i-1,n+1} + u_{i-1,n})$$

(7)

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1. von Rosenberg, D. U., Methods for the Numerical Solution of Differential Equations, American Elsevier Publishing Co., Inc., New York, 1969.

$$- \frac{1}{(\Delta r)^2} k(u_{i,n+\frac{1}{2}})(u_{i,n+1} + u_{i,n}) +$$

$$+ \frac{k(u_{i,n+\frac{1}{2}})}{2(a+(i-\frac{1}{2})\Delta r)} \left(\frac{u_{i+1,n+1} + u_{i+1,n} - u_{i-1,n+1} - u_{i-1,n}}{2\Delta r}\right)$$

$$= c(u_{i,n+\frac{1}{2}})\rho(u_{i,n+\frac{1}{2}}) \frac{u_{i,n+1} - u_{i,n}}{\Delta t} \tag{7}$$

However, the coefficients of the first 2 terms can be viewed as truncated Taylor series for $k(u_{i+\frac{1}{2},n+\frac{1}{2}})$

$$k(u_{i+\frac{1}{2},n+\frac{1}{2}}) = k(u_{i,n+\frac{1}{2}}) + \frac{\Delta r}{2} k'(u_{i,n+\frac{1}{2}}) \left(\frac{u_{i+1,n+\frac{1}{2}} - u_{i-1,n+\frac{1}{2}}}{2\Delta r}\right) \tag{8}$$

Rewriting Eq. (7)

$$\frac{1}{2(\Delta r)^2} [k(u_{i+\frac{1}{2},n+\frac{1}{2}})](u_{i+1,n+1} + u_{i+1,n}) +$$

$$+ \frac{1}{2(\Delta r)^2} [k(u_{i-\frac{1}{2},n+\frac{1}{2}})](u_{i-1,n+1} + u_{i-1,n}) -$$

$$- \frac{1}{2(\Delta r)^2} [k(u_{i+\frac{1}{2},n+\frac{1}{2}}) + k(u_{i-\frac{1}{2},n+\frac{1}{2}})](u_{i,n+1} + u_{i,n}) + \tag{9}$$

$$+ \frac{k(u_{i+\frac{1}{2},n+\frac{1}{2}})}{4(a+(i-\frac{1}{2})\Delta r)} \left(\frac{u_{i+1,n+1} + u_{i+1,n}}{2\Delta r}\right) - \frac{k(u_{i-\frac{1}{2},n+\frac{1}{2}})}{4(a+(i-\frac{1}{2})\Delta r)} \left(\frac{u_{i-1,n+1} + u_{i-1,n}}{2\Delta r}\right) -$$

$$- [c(u_{i+\frac{1}{2},n+\frac{1}{2}}) + c(u_{i-\frac{1}{2},n+\frac{1}{2}})][\rho(u_{i+\frac{1}{2},n+\frac{1}{2}}) + \rho(u_{i-\frac{1}{2},n+\frac{1}{2}})]$$

$$\left[\frac{u_{i,n+1} - u_{i,n}}{\Delta t}\right] = 0$$

where

$$k(u_{i,n+\frac{1}{2}}) = \tfrac{1}{2}[k(u_{i+\frac{1}{2},n+\frac{1}{2}}) + k(u_{i-\frac{1}{2},n+\frac{1}{2}})] \tag{10}$$

Equation (9) is now rewritten for evaluation of temperatures at the $n+\frac{1}{2}$ time interval using for the coefficients their known values at time step n

$$\frac{1}{2(\Delta r)^2}\left[1 + \frac{\Delta r}{4(a+(i-\frac{1}{2})\Delta r)}\right] k(u_{i+\frac{1}{2},n})(u_{i+1,n+\frac{1}{2}} + u_{i+1,n}) + \frac{1}{2(\Delta r)^2}$$

$$\left[1 - \frac{\Delta r}{4(a+(i-\frac{1}{2})\Delta r)}\right] k(u_{i-\frac{1}{2},n})(u_{i-1,n+\frac{1}{2}} + u_{i-1,n}) - \frac{1}{2(\Delta r)^2}*$$

$$[k(u_{i+\frac{1}{2},n}) + k(u_{i-\frac{1}{2},n})](u_{i,n+\frac{1}{2}} + u_{i,n}) - \tfrac{1}{4}[c(u_{i+\frac{1}{2},n}) + c(u_{i-\frac{1}{2},n})]*$$

$$[\rho(u_{i+\frac{1}{2},n}) + \rho(u_{i-\frac{1}{2},n})] \frac{u_{i,n+\frac{1}{2}} - u_{i,n}}{\Delta t} = 0 \tag{11}$$

This yields the temperature distribution at $n+\frac{1}{2}$ using coefficients evaluated from the temperature distribution at time n. The temperatures from Eq. (11) are then used to evaluate the coefficients for use in Eq. (4).

The finite difference equations were tridiagonal and were solved using the Thomas algorithm [1]. Briefly, the form of this algorithm is

$$\begin{aligned} &a_i u_{i-1} + b_i u_i + c_i u_{i+1} = d_i \qquad 1 \leq i \leq R \\ &a_1 = 0, \quad c_R = 0 \end{aligned} \tag{12}$$

where the terms on the left hand side are at the n+1 time step and on the right hand side at n time step. For all i, the quantities

$$\beta_i = b_i - \frac{a_i c_{i-1}}{\beta_{i-1}}, \quad \gamma_i = \frac{d_i - a_i \gamma_{i-1}}{\beta_i} \tag{13}$$

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1. von Rosenberg, D. U. Methods for the Numerical Solution of Differential Equations, American Elsevier Publishing Co., Inc., New York, 1969.

as computed and then back substitution is used to find the temperatures from

$$u_R = \gamma_R$$
$$u_i = \gamma_i - \frac{c_i u_{i+1}}{\beta_i} \tag{14}$$

Computation times are rapid.

IV. THERMAL AND TRANSFORMATION STRESSES. The quenching process gives rise to thermal stresses due to the large thermal gradients that exist. Areas near the boundary are cooler than interior points. The boundary would like to contract but is partially prevented from doing so because of the interior, hence tensile stresses are set up near the boundaries while the interior is in compression. The thermal stresses in an axially symmetric hollow cylinder subject to a non-uniform temperature distribution are given by [2].

$$\sigma_r = \frac{E\alpha}{r^2}\left[\frac{r^2-a^2}{b^2-a^2}\int_a^b \rho u(\rho)d\rho - \int_a^r \rho u(\rho)d\rho\right]$$
$$\sigma_\theta = \frac{E\alpha}{r^2}\left[\frac{r^2+a^2}{b^2-a^2}\int_a^b \rho u(\rho)d\rho - \int_a^r \rho u(\rho)d\rho - r^2 u(r)\right] \tag{15}$$

where σ_r.....the radial stresses
σ_θ.....the tangential stresses
E......Young's Modulus
α......thermal expansion coefficient
$u(\rho)$...radial temperature distribution

The stresses due to the transformation are found using similar equations since these stresses are due mainly to a volume increase in the transformed material. The difference between the two calculations is that the transformation does not occur across the thickness simultaneously but progresses across based on the temperature in the cross sections. No transformation stresses exist when the temperatures are all above that temperature (M_s) when the transformation begins or below that temperature (M_f) for which the transformation ends. Between these two temperatures, a linear change in volume is assumed. The change in volume, about 4% if the transformation is complete, is assumed to be isotropic so that

2. Boley, B. A. and Weiner, J. H., Theory of Thermal Stresses, John Wiley and Sons, Inc., 1960.

it translates to one-third of the volume change for a linear change during the transformation. Stresses are computed in a manner similar to thermal stresses.

$$\sigma_r = \frac{E}{r^2}\left[\frac{r^2-a^2}{b^2-a^2}\int_a^b \frac{\Delta\ell}{\ell}\rho d\rho - \int_a^r \frac{\Delta\ell}{\ell}\rho d\rho\right]$$

$$\sigma_\theta = \frac{E}{r^2}\left[\frac{r^2+a^2}{b^2-a^2}\int_a^b \frac{\Delta\ell}{\ell}\rho d\rho + \int_a^r \frac{\Delta\ell}{\ell}\rho d\rho - r^2\frac{\Delta\ell}{\ell}\right] \qquad (16)$$

where

$$\text{I.} \quad \frac{\Delta\ell}{\ell} = 0 \qquad \text{if } u(r) \geq M_s$$

$$\text{II.} \quad \frac{\Delta\ell}{\ell} = \left(\frac{\Delta\ell}{\ell}\right)^* \frac{1}{M_s-M_f}(M_s-u(r)) \qquad M_s \geq u(r) \geq M_f \qquad (17)$$

$$\text{III.} \quad \frac{\Delta\ell}{\ell} = \left(\frac{\Delta\ell}{\ell}\right)^* \qquad M_f \geq u(r)$$

and $\left(\frac{\Delta\ell}{\ell}\right)^*$ is the linear expansion during a transformation.

The M_s temperature was taken to be 350°F and the M_f temperature 150°F for the computations. Figure 2 shows some temperature distributions which can arise. In the upper figure, no transformation has taken place, hence the transformation stresses are zero. In the lower figure, the transformation is occurring from both the inside and outside radius. In sections indicated by I, corresponding to Equations (17), the transformation has not begun, in sections II, the transformation is progressing and in sections III, the transformation is complete.

As stated above, for the present study the thermal and transformation stresses were assumed to be elastic. In the following, the results indicate that stresses are too large for this assumption to be valid. References [3] and [4] treat similar problems using elastic-plastic analysis.

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3. Weiner, J. H., and Huddleston, J. V., Transient and Residual Stresses in Heat-Treated Cylinders, Journal of Applied Mechanics, March 1959.

4. Landau, H. G., and Zwicky, E. E. Jr., Transient and Residual Thermal Stresses in an Elastic-Plastic Cylinder, Journal of Applied Mechanics, September 1960.

V. RESULTS AND DISCUSSION. Figures 3 through 7 show some resulting temperature distributions under various conditions.

Figure 3 shows the temperature distribution across the wall thickness for various times. The boundary conditions are convective and h_2 represents a value near that suggested by the manufacturer of the quenching facility for the coefficient on the outside diameter. On the inside diameter, the value of h_1 was said to be lower than that of h_2. The radius is in inches and the temperatures are in °F. The ambient temperature was assumed zero. Figure 4 shows the effect of variations in the convection coefficient on the inside diameter. The results are shown for only one time step. Since the temperature at that time does not change much under the different boundary conditions, small differences in the convection parameter on the inside diameter will have little effect on the transformation.

Figure 5 shows the effect when the thermal conductivity is allowed to vary with temperature. For the same time step, three curves are shown. The conductivity is allowed to be an increasing, decreasing or constant function of temperature. Finding real data to use in the program is difficult. Figure 6 shows the temperature distribution for a bilinear thermal conductivity curve based on one for 4130 steel. These properties are usually determined experimentally under equilibrium conditions. Since the structure of the material is changing under rapid cooling and since equilibrium does not exist, the properties which should be used are those determined under the same conditions as the quench. This can be described best by looking at the specific heat. Again for 4130 steel, a spike increase in the value of the specific heat occurs between 1200°F and 1500°F. This occurs during heating and is due to the austenitizing of the material. Figure 7 shows the temperature distribution throughout the tube wall, allowing the specific heat to be a function of the temperature but ignoring the spike mentioned above. Under the quench conditions, the spike would occur during the martensite transformation and be of different shape.

Figures 8 and 9 show a sample of the radial and circumferential or hoop stress, respectively, for a specific time. They are taken from one of the previously cited cases. The insert shows qualitatively the temperature distribution at the time indicating that the transformation has begun at both the inside and outside radius. Each figure shows the thermal, transformation and sum of the stresses. Large compressive hoop stresses indicate the strong possibility of plastic deformation. Since the transformation occurs at the lower temperatures, the thermal gradients are smaller and the thermal stresses lower than their values earlier in the quenching cycle. The thermal stresses, however, can be a significant part of the total stresses, especially early in the transformation, and should not be neglected in an elastic-plastic analysis.

REFERENCES

1. von Rosenberg, D. U., Methods for the Numerical Solution of Differential Equations, American Elsevier Publishing Co., New York, 1969.

2. Boley, B.A. and Weiner, J.H., Theory of Thermal Stresses, John Wiley and Sons, Inc., 1960.

3. Weiner, J. H., and Huddleston, J. V., Transient and Residual Stresses in Heat-Treated Cylinders, Journal of Applied Mechanics, March, 1959.

4. Landau, H. G. and Zwicky, E. E. Jr., Transient and Residual Thermal Stresses in an Elastic-Plastic Cylinder, Journal of Applied Mechanics, September, 1960.

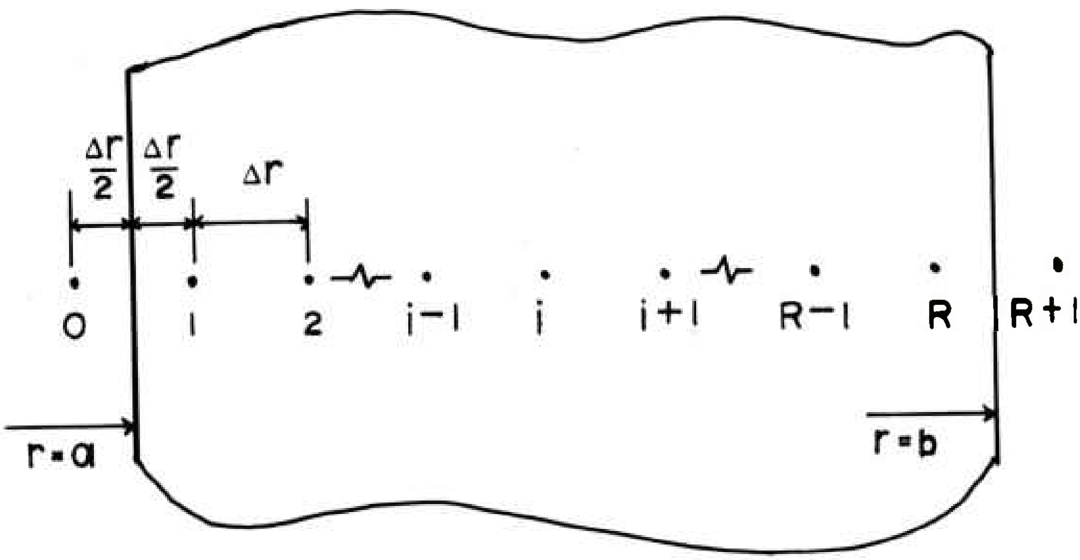

FIGURE 1. NODE PLACEMENT WITH POINTS SHIFTED FROM BOUNDARIES

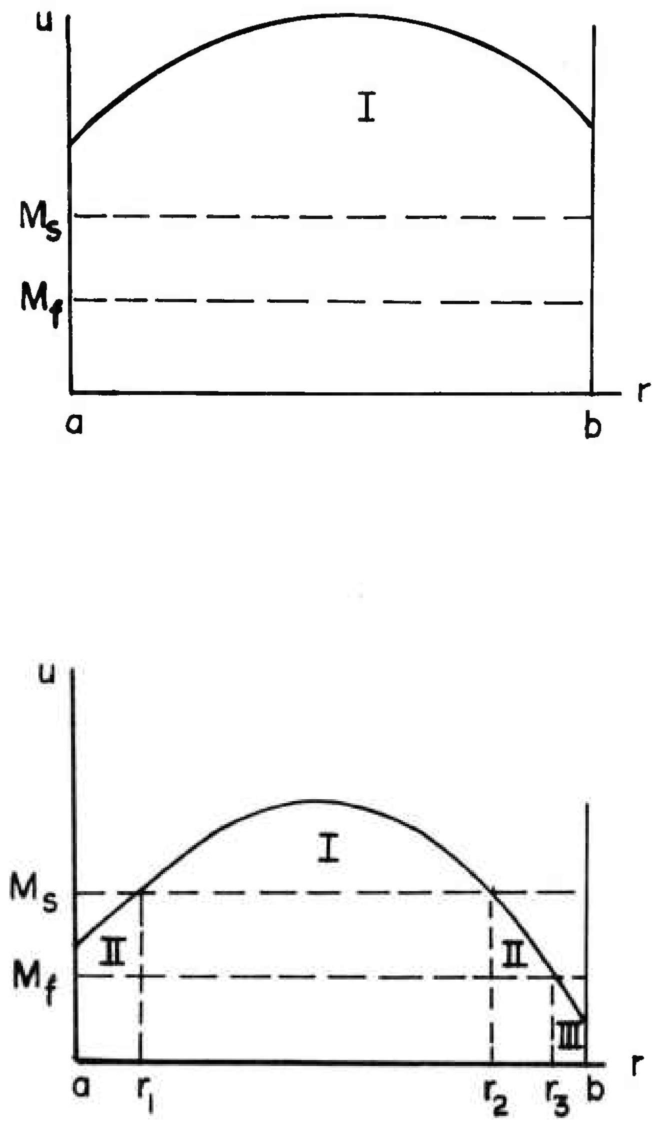

FIGURE 2. TYPICAL TEMPERATURE DISTRIBUTIONS NEAR TRANSFORMATION TEMPERATURE

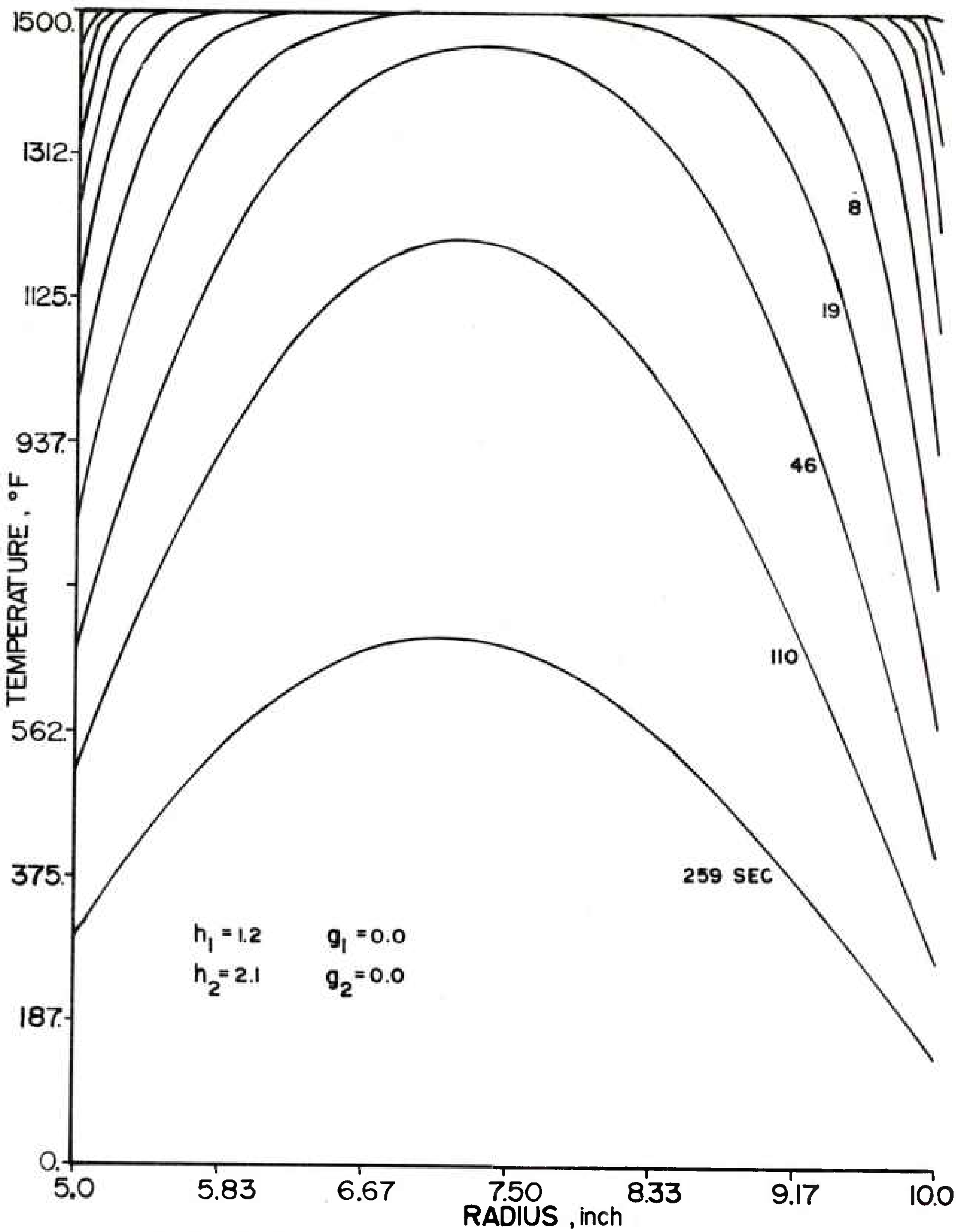

FIGURE 3. TEMPERATURE VS RADIUS FOR VARIOUS TIMES

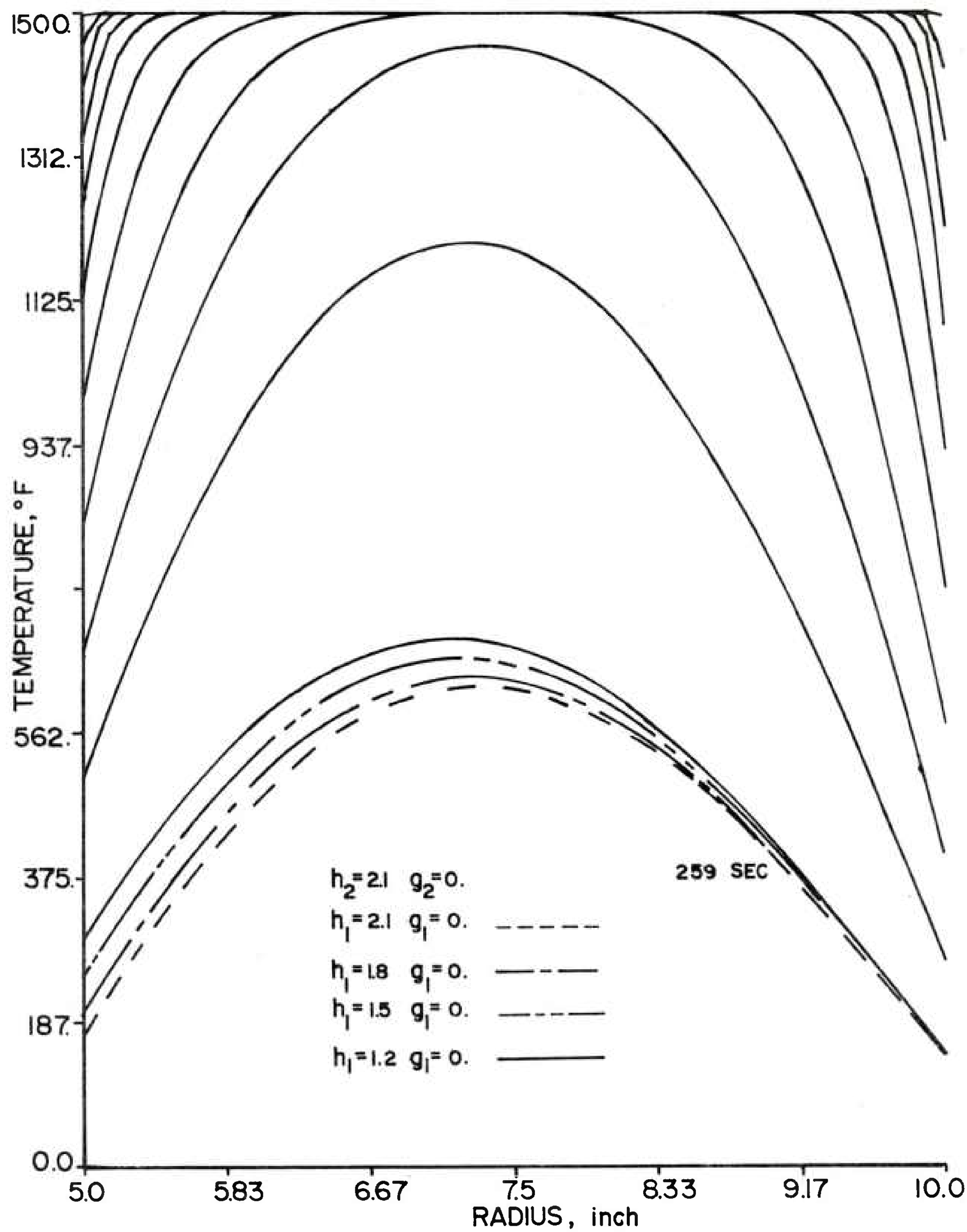

FIGURE 4. VARIATION OF BORE CONVECTION COEFFICIENT

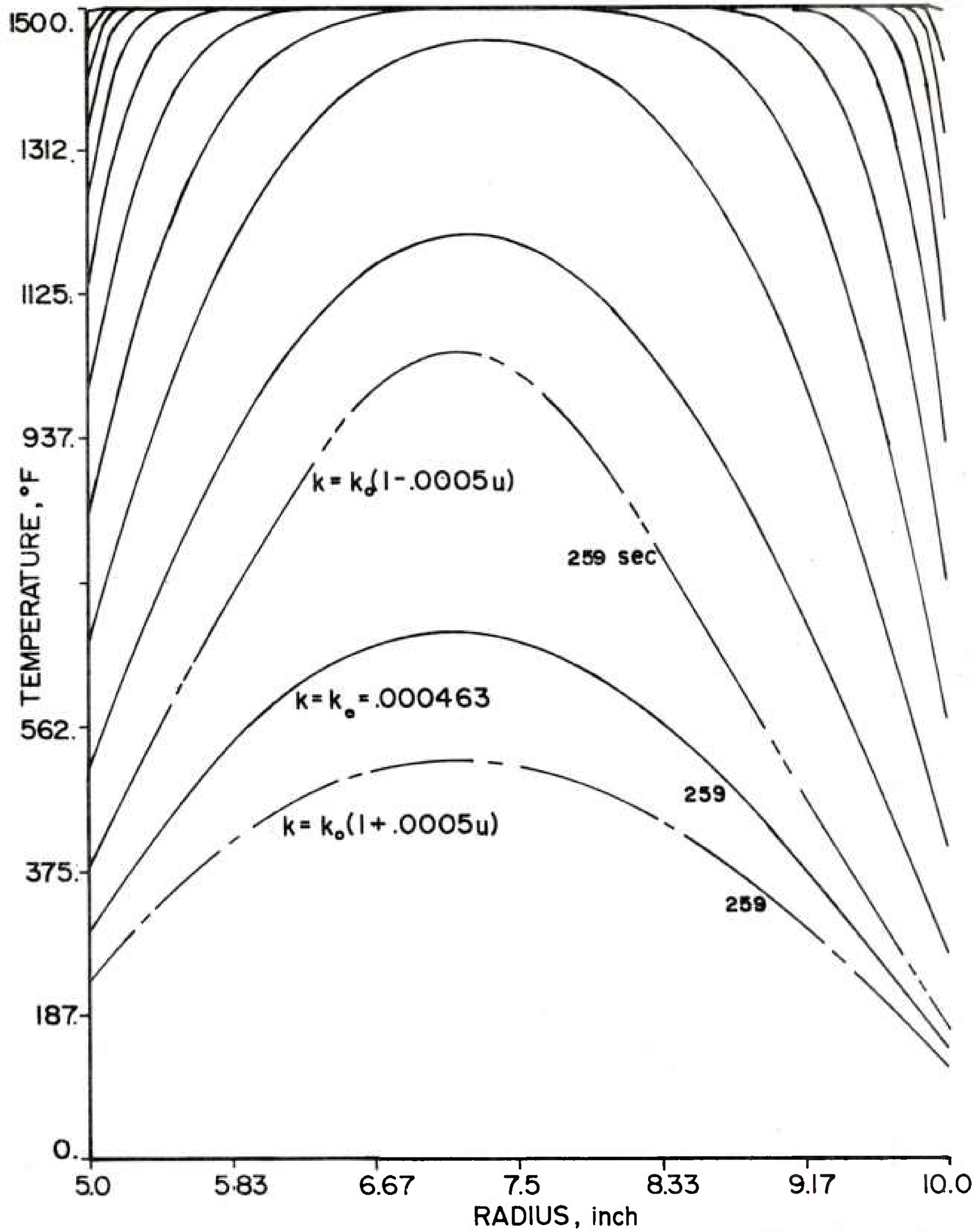

FIGURE 5. VARIATION OF CONDUCTIVITY WITH TEMPERATURE

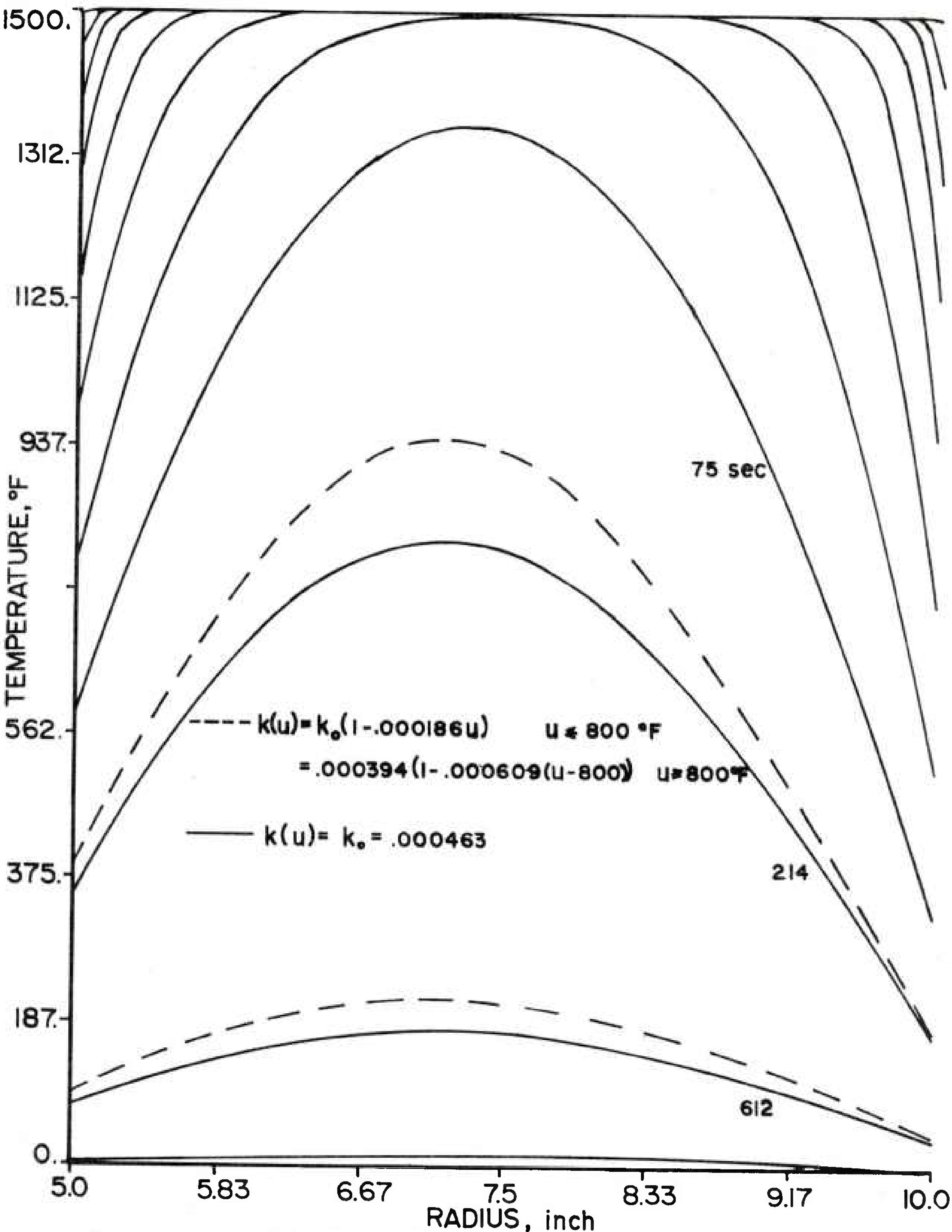

FIGURE 6. VARIATION OF CONDUCTIVITY WITH TEMPERATURE

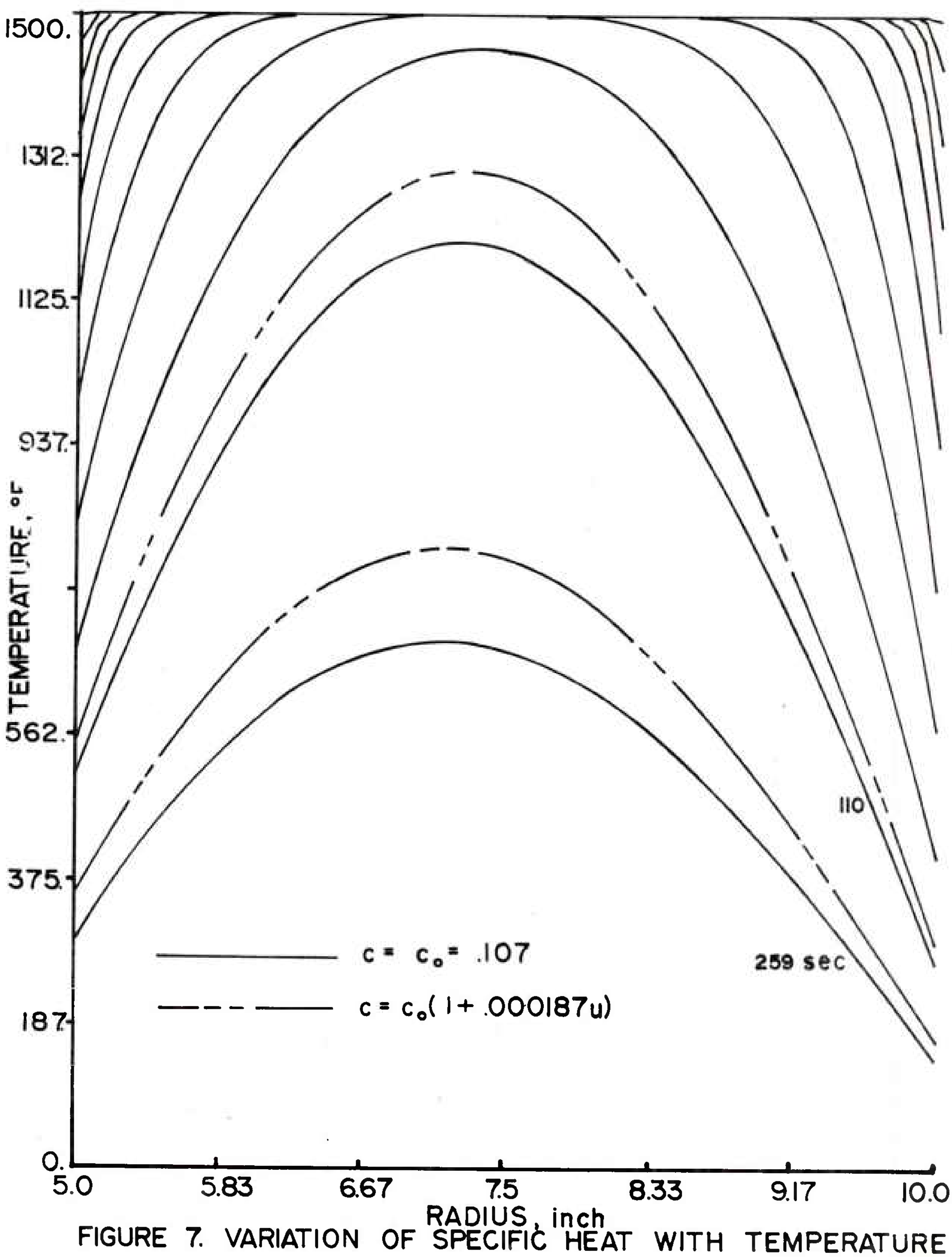

FIGURE 7. VARIATION OF SPECIFIC HEAT WITH TEMPERATURE

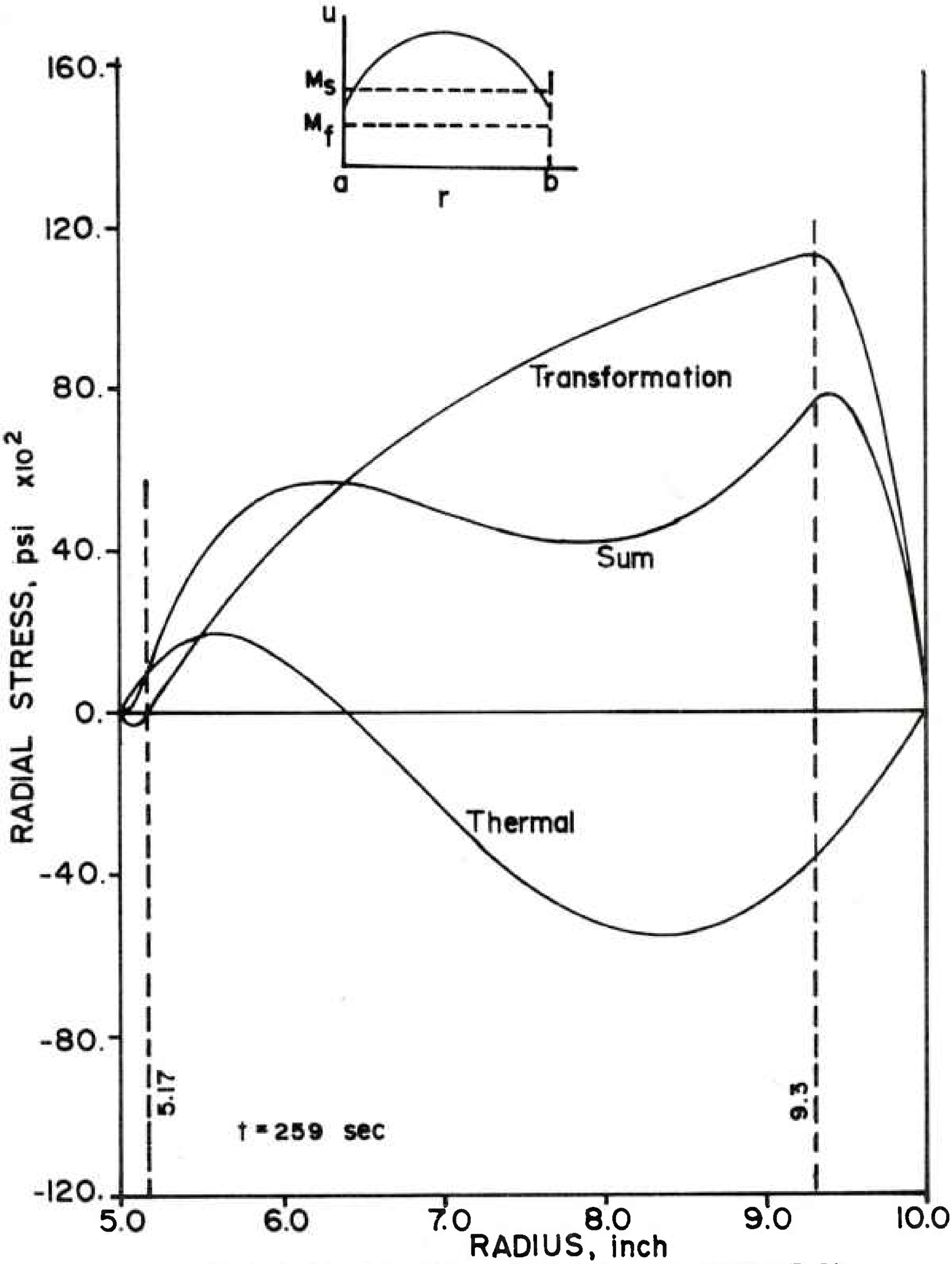

FIGURE 8. RADIAL STRESS DISTRIBUTION

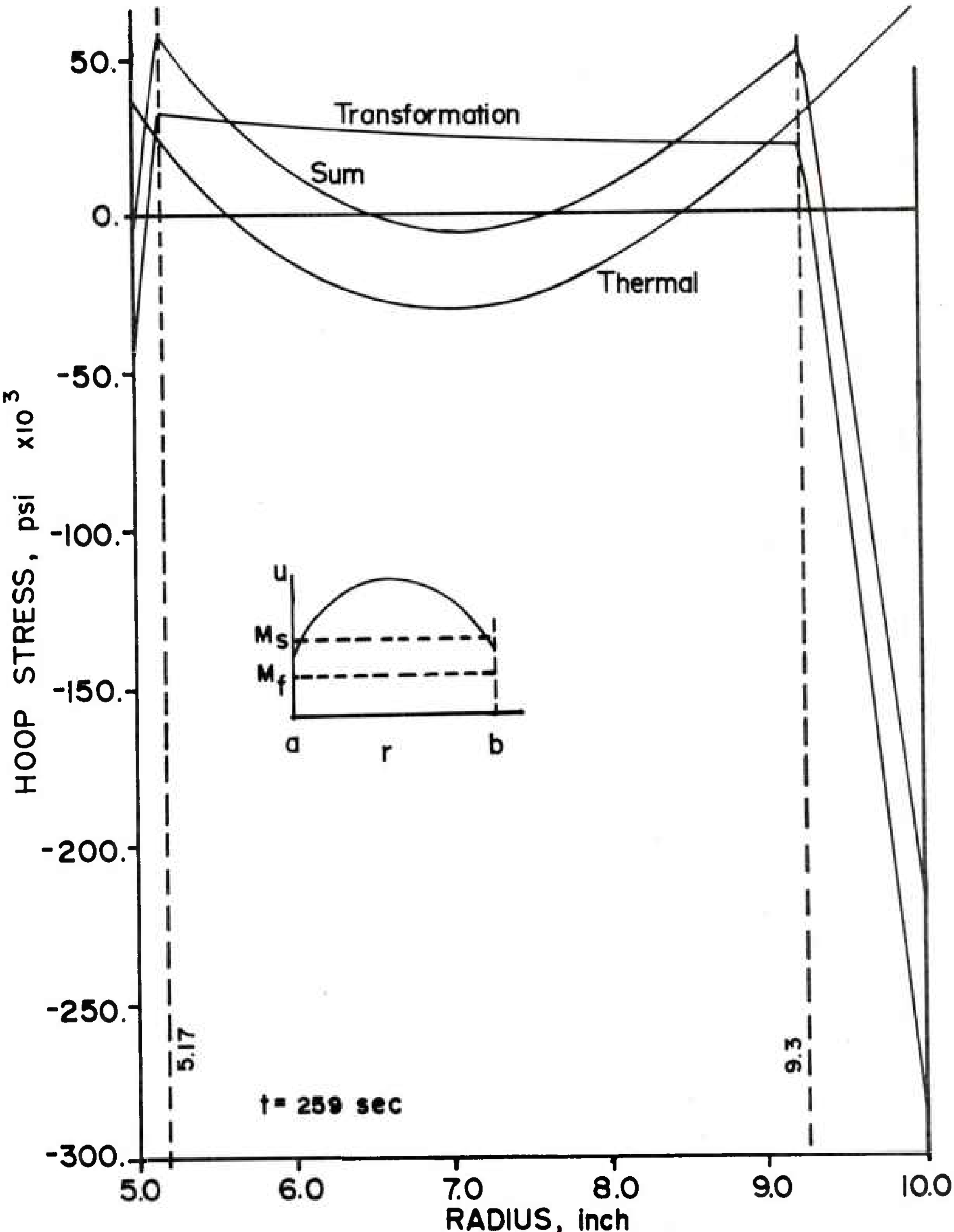

FIGURE 9. HOOP STRESS DISTRIBUTION

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